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Determination of Wavelengths which Correspond to Taken Propagation Constant of Planar Waveguide by Wave Equation Fourier Transform Method

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The paper describes a numerical method based on the Fourier transform application for studying propagating optical waves in dielectric planar waveguides. The inverse problem to known direct one in waveguide investigation is proposed, namely a search of light wavelengths according to taken values of propagation constants. For each constant a set of wavelengths is obtained, among which an input value of wavelength in direct problem exists necessarily. A high accuracy of the method proposed is confirmed by exact values obtained by solution of transcendental dispersion equation. This method is tested on many examples, in particular, for waveguides of different permittivity profiles or modes of TE- and TM-polarization.

Keywords: Fourier transform, Permittivity, Propagation constant, Spatial frequency, Wavelength.

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1. INTRODUCTION

Planar waveguides are the basis of a number of integrated optics elements [1], including distributed feedback microlasers [2, 3]. For device designing based on planar waveguides, it is necessary to know propagation constants of waveguide modes which correspond to taken wavelength. A number of approximate methods are used to determine propagation constants of localized modes of gradient planar waveguides [4], which for the first time have been developed for analysis the tasks of quantum mechanics. A typical permittivity distribution of symmetric gradient waveguide is shown in Fig. 1, where $\lim_{x\to\pm\infty}\varepsilon(x)=\varepsilon_0$, ε_0 is the substrate permittivity, ε_1 is the maximum value of permittivity in active layer. There are found exact analytical solutions for some profiles of waveguide permittivity $\varepsilon(x)$ [1].

If, in this waveguide $\varepsilon_1 > \varepsilon_0$, propagating of localized waveguide mode with propagation constant β is possible, and electric field distribution is described by the following function: $E(x,z) = E(x)\exp(-i\beta z)$, where x and z are the transverse and longitudinal coordinates respectively, $E(x)$ is the electric field amplitude, i is the imaginary unit. But, even in the simplest case a search of propagation constant goes to solution of transcendental algebraic equation. Problem becomes more difficult if permittivity varies according to a complex function along axis x. Well-known methods to find propagation constants and waveguide mode fields are mostly analytical, too cumbersome, and their accuracy is low. These methods have been developed (in quantum mechanics) when computers did not exist or were readily available.

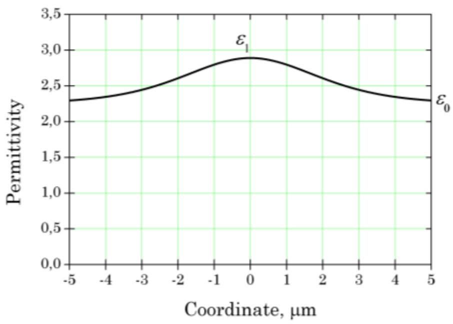

Fig. 1 – An image of permittivity distribution for symmetric planar waveguide

The current state of computer hardware and software sophistication allows use numerical methods to search propagation constants and fields of gradient planar waveguides. It is known a numerical method for finding propagation constants based on wave equation Fourier transform [5], and it is characterized by high accuracy of analysis. By this method it is possible to find all propagation constants of localized modes and appropriate discrete Fourier transforms of field distribution in a waveguide in one calculation cycle. The method is tested for many gradient waveguides. For example, let waveguide permittivity is described by a function $\varepsilon(x) = \varepsilon_0 + (\varepsilon_1 - \varepsilon_0)/\cosh^2(2x/d)$, where d is the thickness of active layer. Then for this waveguide, an exact analytical solution exists, and exact values of propagation constants are found [1], which are listed in the left column of table 1 for a waveguide with the following parameters: $\varepsilon_0 = 2.25$, $\varepsilon_1 = 2.89$, $d = 5$ μm, $\lambda = 1$ μm (light wavelength). At this wavelength waveguide has 13 guided modes. In the right column of the table 1, propagation constants calculated by numerical method described in [5] is shown.

Propagation constants of waveguide modes calculated by both methods are the same, except the last which appropriate field has a maximum length in coordinate space; so for it a small error is present. This numerical method provides high calculation accuracy and, as research shows, it is characterized by high numerical stability. A search of propagation constants by numerical method [5] is reduced to the problem on eigenvalues (square of propagation constants) and eigenvectors (field discrete Fourier transforms in a waveguide) which look like as $\mathbf{MV} = \beta^2\mathbf{V}$.

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Table 1 – Propagation constants of planar gradient waveguide

Index of propagation constant	Propagation constants obtained by exact method [1]	Propagation constants obtained by numerical method [5]
0	10.5905815071	10.5905815071
1	10.4142208649	10.4142208649
2	10.2504427117	10.2504427117
3	10.0998591747	10.0998591747
4	9.9630685490	9.9630685490
5	9.8406460395	9.8406460395
6	9.7331338241	9.7331338241
7	9.6410307334	9.6410307334
8	9.5647819194	9.5647819194
9	9.5047689466	9.5047689466
10	9.4613007717	9.4613007717
11	9.4346060785	9.4346060785
12	9.4248280753	9.4248273946

In practice, it often happens that one needs to solve the inverse problem, i.e., for planar waveguide with certain parameters propagation constant is known, and it is necessary to find wavelengths that correspond to taken propagation constant. This problem arises in the analysis of waveguide distributed feedback lasers by coupled wave method [2]. During the analysis propagation constants at which generation is possible are determined. Let's show that this problem can be solved successfully by numerical method based on wave equation in a frequency domain. The task is reduced again to the eigenvalue/eigenvector problem where square of wavelengths are eigenvalues: $\mathbf{M_1V} = \lambda^2\mathbf{M_2V}$.

2. ONE-DIMENTIONAL WAVE EQUATIONS AND THEIR FOURIER TRANSFORMS

If, in a waveguide mode, electric field is perpendicular to plane xz (TE polarization), wave equation will look like:

$$\frac{d^2E(x)}{dx^2} + \left(\frac{2\pi}{\lambda}\right)^2 \varepsilon(x)E(x) = \beta^2 E(x). \qquad (2.1)$$

If, in a waveguide, TM polarization wave is propagating, appropriate wave equation with regard to magnetic field $H(x)$ can be written as:

$$\frac{d^2H(x)}{dx^2} - \frac{d\ln\varepsilon(x)}{dx}\frac{dH}{dx} + \left(\frac{2\pi}{\lambda}\right)^2 \varepsilon(x)H(x) = \beta^2 H(x). \qquad (2.2)$$

Functions $E(x)$, $H(x)$ describing fields in waveguide localized modes and their first derivatives tend towards zero at $x\to\pm\infty$, and they are absolutely integrated. That is why for these functions, their first and second derivatives the Fourier transform exists. One can write appropriate equations for $E(x)$:

$$E(u) = \int_{-\infty}^{\infty} E(x)\exp(-i2\pi ux)dx, \qquad (2.3)$$

$$i2\pi uE(u) = \int_{-\infty}^{\infty} \frac{dE(x)}{dx}\exp(-i2\pi ux)dx, \qquad (2.4)$$

$$-(2\pi u)^2 E(u) = \int_{-\infty}^{\infty} \frac{d^2E(x)}{dx^2}\exp(-i2\pi ux)dx, \qquad (2.5)$$

where u is the spatial frequency, $E(u)$ is the Fourier transform of electric field.

Besides, for functions for which Fourier transforms exist, i.e., $F\{g(x)\} = G(u)$, $F\{h(x)\} = H(u)$, the next equation is yet right:

$$F\{g(x)h(x)\} = \int_{-\infty}^{\infty} G(u-v)H(v)dv, \qquad (2.6)$$

where $F\{\ldots\}$ is the Fourier transform. Equation (2.6) is named the convolution theorem.

One takes Fourier transforms of left and right parts of (2.1) and (2.2) taking into account (2.3) – (2.6). As a result, we obtain next wave equations in a frequency domain:

$$-4\pi^2u^2E(u) + \left(\frac{2\pi}{\lambda}\right)^2 \int_{-\infty}^{\infty} \varepsilon(u-v)E(v)dv = \beta^2E(u), \qquad (2.7)$$

$$-4\pi^2u^2H(u) - 2i\pi u\int_{-\infty}^{\infty} F\left\{\frac{d\ln\varepsilon(x)}{dx}\right\}(u-v)vH(v)dv +$$

$$+\left(\frac{2\pi}{\lambda}\right)^2 \int_{-\infty}^{\infty} \varepsilon(u-v)H(v)dv = \beta^2H(u). \qquad (2.8)$$

The Fourier transform of permittivity is

$$\varepsilon(u) = \varepsilon_0\delta(x) + (\varepsilon_1 - \varepsilon_0)F\{f(x)\}, \qquad (2.9)$$

where $\delta(x)$ is the Dirac delta function, $f(x)$ is the function describes permittivity distribution.

3. A METHOD TO SEARCH WAVELENGTHS ACCORDING TO TAKEN PROPAGATION CONSTANT

For demonstrating of method to find wavelengths corresponding to taken propagation constant β, let's consider equation (2.7) as easier in the form:

$$4\pi^2\int_{-\infty}^{\infty} \varepsilon(u-v)E(v)dv = \lambda^2\left[\beta^2 + 4\pi^2u^2\right]E(u). \qquad (3.1)$$

In (3.1) one can replace integral by sum and go to the equation in discrete form. By changing continuous values of u and v on discrete ones, we obtain:

$$4\pi^2 \sum_{k=-(N-1)/2}^{(N-1)/2} \varepsilon(s\Delta - k\Delta)E(k\Delta)\Delta =$$

$$= \lambda^2\left[\beta^2 + 4\pi^2(s\Delta)^2\right]E(s\Delta), \qquad (3.2)$$

where N is the number of points in which electric field is sought, s and k are the indices on which summation is done: $s, k \le |(N-1)/2|$, Δ is the partitioning step of maximum spatial frequency u_{max}: $\Delta = u_{max}/N$. Value of N should be taken large enough and unpaired.

One can write the last equation for all discrete spatial frequencies $u_s = s\Delta$. Then a set of these equations will be written in a matrix form where λ^2 is common to all values of index s:

$$\mathbf{M_1V} = \lambda^2\mathbf{M_2V}, \qquad (3.3)$$

where $\mathbf{M_1}$ is the square symmetric matrix of elements $4\pi^2\varepsilon(s\Delta - k\Delta)$, $\mathbf{M_2}$ is the diagonal matrix of elements $\beta^2 + 4\pi^2(s\Delta)^2$, $\mathbf{V}$ is the vector-column of elements $E(s\Delta)$.

So, the problem was led to the problem on eigenvalues (square wavelength) and eigenvectors (discrete Fourier transform of field $E(s)$) which correspond to found value of λ^2. By carrying out the inverse discrete Fourier transform of eigenvector, we obtain field distribution $E(x)$ in a discrete form too.

Propagation constants β_v of symmetric planar waveguide for taken wavelength λ satisfy the following inequality: $2\pi n_0 / \lambda < \beta_v < 2\pi n_1 / \lambda$, where n_0 and n_1 are the refractive indices of substrate and active layer respectively. For the inverse problem wavelengths λ_v must satisfy the following inequality accordingly known propagation constant β:

$$2\pi n_0 / \beta < \lambda_v < 2\pi n_1 / \beta . \qquad (3.4)$$

For all propagation constants from table 1 matching sets of wavelengths are found by usage matrix equation (3.3) and inequality (3.4). In each set a wavelength $\lambda = 1$ μm is present that confirms the correctness of calculations. If, we take arbitrary wavelength from each set and use the equation $\mathbf{MV} = \beta^2\mathbf{V}$, we find appropriate propagation constant among set obtained again. A solution of both direct and inverse problems was carried out at next numerical process parameters: the number of points $N = 2001$, the maximum spatial frequency $u_{max} = 10$ μm^{-1}. They are selected from the subject to the Whittaker-Shannon sampling theorem.

4. CONCLUSIONS

The numerical method to find wavelengths which correspond to taken propagation constant of gradient planar waveguide is developed. The problem is reduced to the higher algebra problem on eigenvalues and eigenvectors like as $\mathbf{M_1V} = \lambda^2\mathbf{M_2V}$. The method provides high accuracy of calculations, and it is characterized by numerical stability.

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